Model-based Monitoring using Dynamic Uncertainty Space Partitioning*

Bernhard Rinner and Ulrich Weiss
Institute for Technical Informatics
Graz University of Technology, AUSTRIA
{rinner, uweiss}@iti.tu-graz.ac.at

Abstract

Monitoring gains importance for many technical systems such as robots, production lines or anti lock brakes. A monitoring system for technical systems must be able to deal with incomplete knowledge of the supervised system, to process noisy observations and to react within predefined time windows.

This paper presents a new approach to monitoring technical systems based on imprecise models. Our approach repeatedly partitions the uncertainty space of an imprecise model and checks the derived model's state for consistency with the measurements. Inconsistent partitions are then refuted resulting in a smaller uncertainty space and a faster failure detection. We demonstrate our monitoring approach on complex heating system where we achieve a significant reduction of the failure detection time compared to conventional interval methods.

Keywords: fault detection; identification; uncertainty space partitioning; self-calibrating monitoring

1 Introduction

The primary objective of a monitoring system is to detect abnormal behaviors of a supervised system as soon as possible to avoid possible shutdown or damage. Technical systems such as robots, production lines or anti lock brakes provide a vast number of challenges for a monitoring system, i.e., it must be able to deal with incomplete knowledge about the supervised system, to process noisy observations and to react within predefined time windows.

*This work has been supported by the Austrian Science Fund under grant number P14233-INF.

A particularly important and widely-applied approach is *model-based monitoring* [3, 2] which relies on a comparison of the predicted behavior of a model with the observed behavior of the supervised system. This paper presents a new approach to monitoring technical systems based on imprecise models. Our approach repeatedly partitions the uncertainty space of an imprecise model and checks the derived model's state for consistency with the measurements. Inconsistent partitions are then refuted resulting in a smaller uncertainty space and a faster failure detection. Our approach is applied to "self-calibrating monitoring" which starts the monitoring process with a large uncertainty space in the model and uses the measurements from the healthy system to refine the intervals of the uncertain parameters resulting in a more precise model.

The remainder of this paper is organized as follows. Section 2 describes the technical details of our approach. Section 3 presents experimental results of our monitoring approach on a complex heating system. A discussion and a summary of related work conclude this paper.

2 Monitoring Based on Uncertainty Space Partitioning

2.1 Overview

Monitoring methods based on imprecise models can reason with incomplete knowledge in the model as well as with noisy measurements. A main drawback of this approach, however, is that the derived envelopes from this imprecise model may diverge very rapidly which delays or even inhibits a failure recognition. We have revised this interval approach to model-based monitoring with the primary goal to keep the resulting envelopes as small as possible.

In our approach, we exploit the measurements from the supervised system as soon as possible to refine the uncertainty in the model and the derived envelopes. The key step in our approach is to partition the uncertainty space of the model into several subspaces. The uncertainty space is given by the bounding intervals on model parameters. The trajectories derived from each subspace are then checked for consistency with the measurements. Each inconsistent subspace is refuted and excluded from further investigations. Partitioning and consistency checking are continued resulting in a smaller uncertainty space of the model. When all subspaces are refuted, a discrepancy between model prediction and observation has been recognized resulting in a fault detection.

2.2 Subspace Partitioning and Consistency Checking

In general, a technical system can be modeled as

$$\begin{aligned} \mathbf{x}_t &= \mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}, \mathbf{p}_{t-1}) \\ \mathbf{y}_t &= \mathbf{g}(\mathbf{x}_t, \mathbf{p}_t) \end{aligned} \tag{1}$$

where $\mathbf{x}_t$ is the state vector at discrete time t, $\mathbf{u}_t$ is the input vector at time t, $\mathbf{p}_t$ is the parameter vector at time t, $\mathbf{y}_t$ is the output vector at time t, and $\mathbf{g}$ and $\mathbf{f}$ are vector functions. In an exact model, $\mathbf{p}_t$ is a vector of real numbers. However, in a model with uncertain parameters, $\mathbf{p}_t$ is replaced by a vector of intervals $\tilde{\mathbf{p}}_t = [(\underline{p}_{1,t}, \overline{p}_{1,t}), (\underline{p}_{2,t}, \overline{p}_{2,t}), \cdots, (\underline{p}_{K,t}, \overline{p}_{K,t})]^T$, where K is the number of uncertain parameters. A model with uncertain parameters, i.e., an *imprecise model*, can therefore be described as:

$$\begin{aligned} \tilde{\mathbf{x}}_t &= \mathbf{f}(\tilde{\mathbf{x}}_{t-1}, \mathbf{u}_{t-1}, \tilde{\mathbf{p}}_{t-1}) \\ \tilde{\mathbf{y}}_t &= \mathbf{g}(\tilde{\mathbf{x}}_t, \tilde{\mathbf{p}}_t) \end{aligned} \tag{2}$$

Equation 2 is the starting point of our approach. It defines an imprecise model of the supervised system with K uncertain parameters. Thus, this model has a K-dimensional uncertainty space. In order to divide this uncertainty space we have to define a *partition* $\tilde{\mathbf{q}}_t = [(\underline{q}_{1,t}, \overline{q}_{1,t}), (\underline{q}_{2,t}, \overline{q}_{2,t}), \cdots, (\underline{q}_{K,t}, \overline{q}_{K,t})]^T$ with $\tilde{\mathbf{q}}_t \subseteq \tilde{\mathbf{p}}_t$. A complete partitioning of the uncertainty space at any time t into M partitions must satisfy the following condition $\bigcup_m \tilde{\mathbf{q}}_t^{(m)} = \tilde{\mathbf{p}}_t$ where $m = 1, \ldots, M$. A model based on a partition of the uncertainty space is referred to as *subspace*. From the definition of a partition, we can finally define the state of a subspace m:

$$\begin{aligned} \tilde{\mathbf{x}}_t^{(m)} &= \mathbf{f}(\tilde{\mathbf{x}}_{t-1}^{(m)}, \mathbf{u}_{t-1}, \tilde{\mathbf{q}}_{t-1}^{(m)}) \\ \tilde{\mathbf{y}}_t^{(m)} &= \mathbf{g}(\tilde{\mathbf{x}}_t^{(m)}, \tilde{\mathbf{q}}_t^{(m)}). \end{aligned} \tag{3}$$

With the monotonicity assumption of $\mathbf{y}$ with regard to the parameters $\mathbf{p}_t$ over the range of the intervals, the (uncertain) state of a subspace can be represented by the (exact) state of the *corner points* of a subspace. The corner points of a subspace are defined as all combinations of upper and lower bounds of a partition $\tilde{\mathbf{q}}_\mathbf{t}$ and can be represented as set $Q_t^{(m)} = \{\tilde{\mathbf{q}}_{t,i}^{(m)}\}$ with $i = 1, \ldots, 2^K$. Thus, an uncertainty space of dimension K results in 2^K corner points. The states at the corner points can be represented as set

$$\begin{aligned} \mathbf{X}_t^{(m)} &= \{\mathbf{x}_{t,i}^{(m)} : \mathbf{x}_{t,i}^{(m)} = \mathbf{f}(\mathbf{x}_{t-1,i}^{(m)}, \mathbf{u}_{t-1}, \mathbf{q}_{t-1,i}^{(m)})\} \quad i = 1, \ldots, 2^K \\ \mathbf{Y}_t^{(m)} &= \{\mathbf{y}_{t,i}^{(m)} : \mathbf{y}_{t,1}^{(m)} = \mathbf{g}(\mathbf{x}_{t,i}^{(m)}, \mathbf{q}_{t,i}^{(m)})\} \quad i = 1, \ldots, 2^K \end{aligned} \tag{4}$$

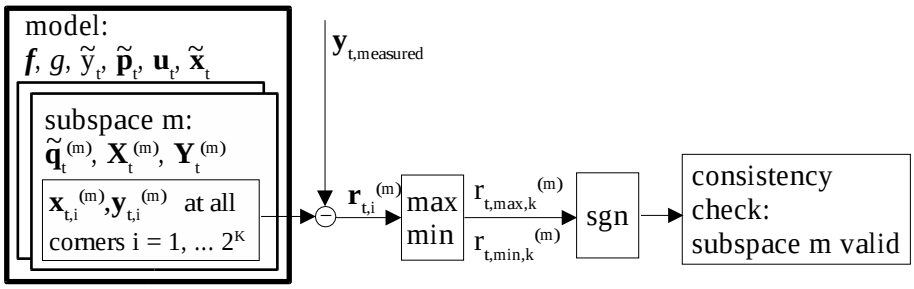

Figure 1: Overview of monitoring based on subspace partitioning and consistency checking. Consistency checking is performed for every valid subspace at each discrete time point t.

where $\mathbf{q}_{t,i}^{(m)}$ is an exact parameter vector at time t from the subspace m and at corner i of this subspace. Note, that $\mathbf{x}_{t,i}^{(m)}$ are state vectors, and also $\mathbf{y}_{t,i}^{(m)}$ are output vectors with exact values.

This representation of an uncertain state is directly exploited by our consistency check for a given subspace m. First, a K-dimensional *residual* is calculated for each state at a corner point using the measurements at time t, i.e., $\mathbf{r}_{t,i}^{(m)} = \mathbf{y}_{t,measured} - \mathbf{y}_{t,i}^{(m)}$. Then, the minimum and maximum values of the residual are determined as $r_{t,min,k}^{(m)} = \min_i\{r_{t,i,k}^{(m)}\}$ and $r_{t,max,k}^{(m)} = \max_i\{r_{t,i,k}^{(m)}\}$ with $i = 1, \ldots, 2^K$, and $k = 1, \ldots, K$. Finally, subspace m is checked for consistency simply by comparing the signs of $r_{t,min,k}^{(m)}$ and $r_{t,max,k}^{(m)}$. The subspace m is consistent with the measurements, iff

$$\text{sgn}(r_{t,min,k}^{(m)}) \neq \text{sgn}(r_{t,max,k}^{(m)}) \tag{5}$$

holds for all elements $k = 1, \ldots, K$. Informally, Equation 5 checks whether the zero vector lies within the "residual subspace". If this equation is violated, the subspace m is refuted. This simple consistency check holds also if not all elements of $\mathbf{y}$ are included in the measurements. In this case, a comparison with the missing elements is simply ignored.

Figure 1 depicts the basic steps of subspace partitioning and consistency checking of each subspace. Since this technique is based on the calculation of an exact state (at corner points), we can use standard numerical methods for computing the solution of differential equations.

2.3 Dynamic Partitioning

During the monitoring process many subspaces may be detected as inconsistent with the measurements. To further refine the uncertainty space and, thus, improve the fault detection partitioning, subspace partitioning can be continued for the remaining consistent partitions.

This *dynamic* partitioning is driven by the measurements from the supervised system resulting in smaller subspaces that are describing the supervised system potentially more precisely.

Dynamic partitioning can be performed at any time during the monitoring process. To keep the computation simple we apply it only at discrete sampling time points. For each subspace at time t, we need to know the parameter vectors of the corner points $\tilde{\mathbf{q}}_t$, and the state vectors for all corner points $\mathbf{x}_t$ (refer to Equation 3). Note that each dynamically generated partitioning must also be complete such that $\bigcup_m \tilde{\mathbf{q}'}_t^{(m)} = \tilde{\mathbf{q}}_t$ for $m = 1, \ldots M'$ where M' is the number of subspaces generated from partition $\tilde{\mathbf{q}}_t$ at time t. A simple approach to achieve a complete partitioning is to divide each parameter of the uncertainty space equally into two intervals resulting in 2^K subspaces.

Dynamic partitioning requires the computation of the state vectors of the introduced corner points. A simple approach for the computation of the state vector is to start the calculation at the initial state of these corner points (which are known) and recompute the state vector until the current time point t. This requires all measurements from the initial time to the current time t.

2.4 Time Variance of Parameters

We now relax the assumption that the parameters do not change over time. As a consequence, a partition $\tilde{\mathbf{q}}_\mathbf{t}$ of the uncertainty space can also vary over time. Since we are only interested in the corner points of a partition, we can model the time variance with a limit on the growth rate $\acute{p}_k$ for each parameter p_k.

Thus, a partition can be bound over time as follows:

$$\underline{q}_{k,t+1} = \underline{q}_{k,t} - \Delta t \cdot \acute{p}_k \tag{6}$$

$$\overline{q}_{k,t+1} = \overline{q}_{k,t} + \Delta t \cdot \acute{p}_k \tag{7}$$

with Δt as the time (sampling) period. This is the largest possible partition at time $t+1$, i.e., if the parameter p_k lies between $\underline{q}_{k,t}$ and $\overline{q}_{k,t}$ at time t, the parameter will be bounded by $\underline{q}_{k,t+1}$ and $\overline{q}_{k,t+1}$ at time $t+1$.

The initial bounds on the parameters may provide an additional bound on the partition. If we assume that these bounds are time-invariant, i.e., $\underline{\mathcal{P}}_k \leq q_{k,t} \leq \overline{\mathcal{P}}_k$ with $\underline{\mathcal{P}}_k$ as the lower

initial bound on the parameter p_k and $\overline{\mathcal{P}}_k$ as the upper initial bound on p_k, the growth of a partition is limited by $\underline{\mathcal{P}}_k$ and $\overline{\mathcal{P}}_k$, respectively.

This approach to time-variant parameters requires only a minor modification to our monitoring method. At each new time step, the partitions of all valid subspaces must be extended, i.e., the corner points must be calculated using Equations 6 and 7.

Now this extension makes noise as an additional uncertainty parameter feasible. For noise, the limit of the growth rate $\acute{p}_k$ is ∞, and $\tilde{q}_k$ is set to $[\underline{\mathcal{P}}_k, \overline{\mathcal{P}}_k]$ at each new time step.[1]

3 Experimental Results

We demonstrate the performance of our monitoring algorithm on a "real" technical system which is compromised of three heating/cooling components mounted on a thermal conductive plate. A process control computer (B&R 2003) controls the three heating/cooling components. The measured samples as well as the control actions issued are transferred to the monitoring system via a RS 232 interface.

Our model which includes the three components with heating elements is given as

$$\begin{aligned}
\dot{T}_1 &= \tfrac{1}{C_1}(q_{i1} - L_1(T_1 - T_0) - L_{12}(T_1 - T_2)) \\
\dot{T}_2 &= \tfrac{1}{C_2}(q_{i2} + L_{12}(T_1 - T_2) - L_2(T_2 - T_0) - L_{23}(T_2 - T_3)) \\
\dot{T}_3 &= \tfrac{1}{C_3}(q_{i3} + L_{23}(T_2 - T_3) - L_3(T_3 - T_0))
\end{aligned} \tag{8}$$

where T_i is the temperature of the three components, C_i is the mass of the components, q_i is the heat flow into the components, L_i the thermal conductivity between the component i and the environment, L_{ij} the thermal conductivity between the component i and j, and T_0 the temperature of the environment. We can reduce the complexity of this model by exploiting the symmetric construction of the heating system ($L_3 = L_1$, $L_{23} = L_{12}$, $C_3 = C_1$) resulting in a total of five uncertain parameters.

To get the time-discrete solution of the differential equation, we use the numeric approximation method of Runge-Kutta. The state vector is given as $\mathbf{x} = (T_1, T_2, T_3)^T$, the input vector

[1] This method is not an exact method to handle systems with time-invariant parameters. With this definition we assume a discrete change of the parameters, and the model calculation (Equation 1) is based on the assumption that all $\dot{p}_k$ equals 0. To extend the model description to $\dot{p}_k \neq 0$, terms with the first derivation of the parameters have to be included in the system.

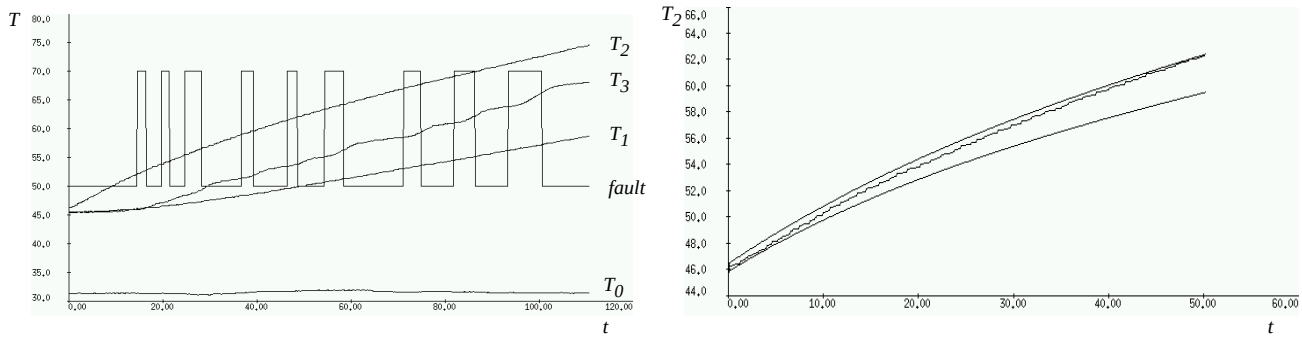

Figure 2: Scenario of an intermittent fault (left) and fault detection by observing temperature sensor T2 (right). The sensor readings of all four temperature sensors as well as the induced fault are plotted in the left graph. The derived trajectories and the sensor reading for temperature sensor T2 are plotted in the right graph.

as $\mathbf{u} = (q_{i1}, q_{i2}, q_{i3}, T_0)^T$, and the output vector as $\mathbf{y} = (T_1 + n_1, T_2 + n_2, T_3 + n_3)^T$, where n_i is the noise of each temperature sensor. The noise parameters are also included in the uncertainty space resulting in a total of eight uncertain parameters. Note that noise parameters are not dynamically partitioned into smaller intervals.

We have measured the input values with $q_{i1} = q_{i3} = 1.24W$ and $q_{i2} = 32.8W$ (heating element 2 is switched on, heating elements 1 and 3 are turned off). In our experiment, we start to refine the interval of the parameters, which are initially given as $L_1 = [0.01, 1]$, $L_2 = [0.01, 1]$, $L_{12} = [0.01, 10]$, $C_1 = [5, 200]$, $C_2 = [5, 200]$. The noise parameters are set to $n_1 = n_2 = n_3 = [-0.3, +0.3]$. After dynamic partitioning has been applied with measurements from the nominal system, the following intervals have been achieved: $L_1 = [0.12, 0.13]$, $L_2 = [0.15, 0.18]$, $L_{12} = [0.62, 0.73]$, $C_1 = [51, 54]$, $C_2 = [61, 65]$. This corresponds to a reduction of the size of the uncertainty space of 8 orders of magnitude.[2]

We demonstrate the fault detection performance using an intermittent fault scenario in component 3 of the heating system (Figure 2 (left)). At $t = 0$ the heating element of component 2 is switched on; all other actuators remain turned off. Starting at $t = 14.6s$, the heating element of component 3 is switched on and off several times. Figure 2 (right) depicts the situation of detecting this fault scenario by observing only the temperature sensor of component 2. At $t = 50s$, the sensor value exceeds the trajectory derived from the imprecise model.

Table 1 presents the time required by our monitoring system for detecting the intermittent

[2]The size of the uncertainty space is defined as the product of the range of all uncertain parameters.

observ. variables	*sampling rate [Hz]*	*fault recognition [s]*			
		model #1	model #2	model #3	model #4
T1, T2, T3	0.1	15.4	5.4	5.4	5.4
	1	5.4	4.4	4.4	4.4
	10	3.8	3.8	3.6	3.3
T2	0.1	> 96.1	55.4	55.4	45.4
	1	94.4	57.4	46.4	38.4
	10	92.5	42.3	33.9	35.6
T1	0.1	> 96.1	> 96.1	> 96.1	> 96.1
	1	> 96.1	> 96.1	> 96.1	> 96.1
	10	> 96.1	> 96.1	> 96.1	> 96.1

Table 1: Time required to detect the intermittent fault using dynamic subspace partitioning.

fault in the heating system. This table summarizes the results from experiments where the uncertainty space of the model, the number of observed variables and the sampling rate have been varied. Model #1 has the largest uncertainty space and model #4 has the smallest uncertainty space. Note that observing only T1 is not sufficient in order to detect the fault within the observation period of $110.7s$.

4 Discussion

In this paper, we have presented a model-based monitoring approach based on uncertainty space partitioning. Our approach repeatedly partitions the uncertainty space of an interval model and checks the derived model's state for consistency with the measurements. Our approach has been implemented to monitor a complex heating system [9].

The main contributions of this approach can be summarized as follows: First, subspace partitioning and consistency checking reduces the detection time for abrupt as well as incipient faults compared to conventional interval methods. Second, the refutation of subspaces that are inconsistent with the measurements can also be seen as *system identification*. As more measurements are available the uncertainty space is reduced resulting in smaller bounding intervals on the parameters. However, our approach is in contrast to traditional system identification where the model space is specified by a parameterized differential equation. Identification selects numerical parameter values so that simulation of the model best matches the measurements. By using refutation instead of search our method is able to derive *guaranteed* bounds on the trajectories. Third, our approach simply accounts for noise in the measurements by introducing

additional uncertain parameters into the model. Finally in our approach, an implementation with differential equations (instead of discrete-time equations) is also possible. A variation of the structure of the model then is easy to make, and it is not necessary, to solve the new model to discrete-time equations. Drawback of this option is the large increase of calculation time when using a high-precise calculation method like Runge-Kutta.

This work is related to the interval identification algorithm of Schaich et al. [8]. In their approach the consistency check is only performed at the qualitative level. Thus, valuable detection time is lost, as long as the fault is only manifested in a quantitative value. Petridis and Kehagias [7] have also developed an algorithm with subspace partitioning. The partitioning is only performed in advance and the consistency check is based on probabilities depending on the noise in the system. Other work in monitoring [4, 6, 1] uses multiple models for failure detection. These models represent known failures of the supervised system. From the viewpoint of system identification, our approach is closely related to semi-quantitative system identification [5]. Identification of both approaches are grounded on the refutation of subspaces that are known to be inconsistent with the measurements. Semi-quantitative system identification performs refinement at the qualitative and interval level.

The granularity of the subspace partitioning has a great influence on the performance of our monitoring approach. If the granularity increases, i.e., the uncertainty space is divided into many but small partitions, the failure detection time decreases and the computation requirements increases and vice versa. For real-time monitoring, subspace partitioning and consistency checking has to keep pace with the temporal evolution of the supervised system, i.e., they must be computed within the sampling period. The calculation of subspaces is independent of each other and can, therefore, be executed in parallel. Thus, if more computing power is required our algorithm can be directly mapped onto a multi-processor system.

Dynamic partitioning is an important feature of this approach, because it limits the envelopes of the output vector. To dynamically partition the uncertainty space, the state vectors of the introduced corner points are required. We currently use the initial state and the complete history of input vectors to calculate these new state vectors. A different approach we are currently investigating is to estimate these state vectors with the inverse function $\mathbf{g}^{-1}$: $\tilde{\mathbf{x}}_t^{(m)} = \mathbf{g}^{-1}(\mathbf{y}_t^{(m)}, \tilde{\mathbf{q}}_t^{(m)})$, by using the measurement vector $\mathbf{y}_{t,measured}$.

Further directions for future work include (i) extending our approach to models that exhibit continuous and discrete behaviors, i.e., hybrid systems, and (ii) applying our approach to different monitoring domains.

References

[1] S. Bogh. Multiple Hypothesis-Testing Approach to FDI for the Industrial Actuator Benchmark. *Control Eng. Practice*, 3(12):1763–1768, 1995.

[2] Jie Chen and Ron J. Patton. *Robust Model-Based Fault Diagnosis for Dynamic Systems.* Kluwer, 1999.

[3] Walter Hamscher, Luca Console, and Johan de Kleer, editors. *Readings in Model-based Diagnosis.* Morgan Kaufmann, 1992.

[4] Peter D. Hanlon and Peter S. Maybeck. Multiple-Model Adaptive Estimation using a Residual Correlation Kalman Filter Bank. *IEEE Transactions on Aerospace and Electronic Systems*, 36(2):393–406, 2000.

[5] Herbert Kay, Bernhard Rinner, and Benjamin Kuipers. Semi-Quantitative System Identification. *Artificial Intelligence*, 119(1-2):103–140, May 2000.

[6] Raman Mehra, Constantino Rago, and Sanjeev Seereeram. Autonomous Failure Detection, Identification and Fault-tolerant Estimation with Aerospace Applications. *Proceedings of the IEEE Aerospace Applications Conference*, 2:133–138, 1998.

[7] V. Petridis and Ath. Kehagias. A Multi-model Algorithm for Parameter Estimation of Time-varying Non-linear Systems. *Automatica*, 34(4):469–475, 1998.

[8] David Schaich, Rudibert King, Uwe Keller, and Mike Chantler. Interval Identification - a Modelling and Design Technique for Dynamic Systems. In *QR99,* 13^{th} *International Workshop on Qualitative Reasoning*, June 6-9 1999.

[9] Siegbert D. Waldner. Implementation of Monitoring System for Technical Systems using imprecise Models. Master's thesis, Graz University of Technology, 2001.